Chapter 2 - Day 1

Calculus describes how quantities change.

Ex: If we drive 210 miles in 3 hours, then our average velocity/speed, or rate of travel is

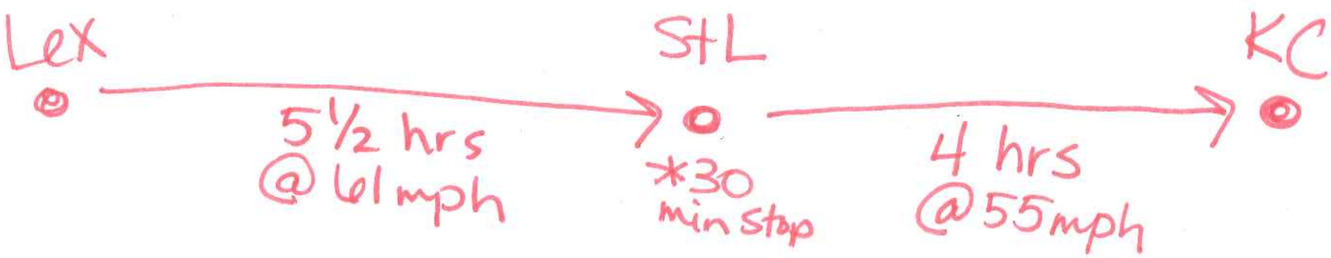
$$210/3 = 70 \text{ mph}$$

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}} = \frac{\Delta s}{\Delta t}$$

Ex: You drive to St. Louis from Lexington which is a 337 mile trip. The drive takes you $5\frac{1}{2}$ hours. What is the average velocity of your car?

$$\frac{337}{5.5} = 61.2\overline{727} \approx 61 \text{ mph}$$

Ex! You drive to St. Louis in $5\frac{1}{2}$ hours averaging 61mph. After a 30 minute snack break, You drive 4 hours & averaging 55mph on our way to Kansas City. How fast did your car average all day (include the stop)?



total time: $5\frac{1}{2} + \frac{1}{2} + 4 = 10$ hrs.

total distance: $d = r \cdot t$

$$\left.\begin{aligned} D_{L\to S} &= (61)(5\tfrac{1}{2}) = 335.5 \\ D_{S\to K} &= (55)(4) = 220 \end{aligned}\right\} \begin{array}{l} 555.5 \\ \text{total miles.} \end{array}$$

$$\text{average velocity} = \frac{\text{miles}}{\text{time}} = \frac{555.5}{10} = 55.55 \text{ mph}$$

The <u>average rate of change</u> of a function $y=f(x)$ is

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

* average rate of change is also the Slope of the secant line between the Points $(x_1, f(x_1))$ and $(x_2, f(x_2))$

<u>Ex</u>: let $g(x) = 3 + 2(x-1)$. Find the average rate of change between $x=1$ and $x=4$.

$g(1) = 3+2(1-1) = 3+2(0) = 3$

$g(4) = 3+2(4-1) = 3+2(3) = 9$

* two points $(1,3)$ and $(4,9)$

$$\text{ARoC} = \frac{\Delta y}{\Delta x} = \frac{9-3}{4-1} = \frac{6}{3} = \boxed{2}$$

* Note: Since ARoC is the slope of the secant line.

$$g(x) = 3 + 2(x-1) = 3 + 2x - 2$$
$$= \underline{\underline{2}}x + 1$$

$\to m = 2 = $ ARoC

Ex: let $f(x) = \sqrt{2x+5}$. Find the ARoC as x goes from -2 to 2

$f(-2) = \sqrt{2(-2)+5} = \sqrt{-4+5} = \sqrt{1} = 1$ $\underline{\underline{(-2,1)}}$

$f(2) = \sqrt{2(2)+5} = \sqrt{4+5} = \sqrt{9} = 3$ $\underline{\underline{(2,3)}}$

$$\text{ARoC} = \frac{\Delta y}{\Delta x} = \frac{3-1}{2-{-2}} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

Ex: Find the average velocity of a moving train whose position is given by $g(t) = 4t^2 + 3t$ as t changes from 0 to 5 seconds.

$$g(0) = 4(0^2) + 3(0) = 0$$

$$g(5) = 4(5^2) + 3(5) = 115$$

$$\frac{\Delta y}{\Delta x} = \frac{115 - 0}{5 - 0} = \frac{115}{5} = \boxed{23}$$

Ex: find ARoC of $k(x) = x^2 + 1$ as x ranges from 1 to $1+h$.

$$\text{ARoC} = \frac{k(1+h) - k(1)}{(1+h) - 1} = \frac{k(1+h) - k(1)}{h}$$

$$= \frac{[(1+h)^2 + 1] - [1^2 + 1]}{h}$$

$$= \frac{1 + 2h + h^2 + 1 - 2}{h} = \frac{2h + h^2}{h}$$

$$= \frac{\cancel{h}(2+h)}{\cancel{h}} = \boxed{2+h}$$

Ex: let $g(x) = \frac{1}{x}$. Find an x such that the average rate of change from 1 to x equals $\frac{-1}{10}$.

$$g(1) = \frac{1}{1} = 1$$

$$\frac{\Delta y}{\Delta x} = \frac{g(x) - g(1)}{x - 1} = \frac{\frac{1}{x} - 1}{x - 1} \cdot \frac{(x)}{(x)}$$

$$= \frac{1 - x}{(x)(x-1)} (-1) = \frac{-1}{x}$$

we want $\frac{-1}{x} = \frac{-1}{10}$

thus $\boxed{x = 10}$